RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2015-18]

B.A./B.Sc. THIRD SEMESTER (July – December) 2016

Mid-Semester Examination, September 2016

Date : 10/09/2016 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper : III Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(***Answer any five questions***) [5×5]

1. Let U and W be two subspaces of a finite dimensional vector space V over a field F. Then prove that $\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$. [5]

2. Let U be the subspace of $\mathbb{R}^4$ generated by $\{(1,0,1,1),(1,2,1,0)\}$. Find two different subspaces P, Q of $\mathbb{R}^4$ such that $U \oplus P = \mathbb{R}^4$ and $U \oplus Q = \mathbb{R}^4$. [5]

3. Let A, B be two matrices over the same field F such that AB is defined. Then show that rank of $AB \le \min\{\text{rank } A, \text{ rank } B\}$

   For what values of 'a' the following system is consistent? [3+2]

   $x - y + z = 1$

   $x + 2y + 4z = a$

   $x + 4y + 6z = a^2$

4. Find the rank of $A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 3 & 1 & 2 & 3 \\ 1 & 7 & 2 & 1 \end{pmatrix}$. Also express the matrix, as a product of two matrices with rank 'rank A'. Find all 1×3 real matrices B such that BA = O where $A = \begin{pmatrix} 1 & -1 & 1 \\ 3 & 2 & 2 \\ 4 & 1 & 3 \end{pmatrix}$. [3+2]

5. For what values of 'k' the planes

   $x + y + z = 2$

   $3x + y - 2z = k$

   $2x + 4y + 7z = k + 1$

   form a triangular prism. Let V and W are vector spaces over the same field F. Let T be a linear transformation from V to W. Then show that ImT is a subspace of W. [3+2]

6. Find the values of 'a' and 'b' for which the system

   $x + y + z = b$

   $2x + y + 3z = b + 1$

   $5x + 2y + az = b^2$

   has (a) only one solution, (b) no solution, (c) many solutions. [5]

7. Let V and W be vector spaces over the same field F. Also let $T : V \to W$ be a linear transformation. If V is finite-dimensional, then show that $\text{nullity}(T) + \text{rank}(T) = \dim V$. [5]

8. Let V be the vector space of all real polynomials of degree ≤ 3 and $D:V\to V$ is defined by $Dp(x)=\frac{d}{dx}p(x)$, $p(x)\in V$ and $T:V\to V$ is defined by $Tp(x)=x\frac{d}{dx}p(x)$, $p(x)\in V$. Relative to the ordered basis $\{1,x,x^2,x^3\}$ determine the matrix of the linear transformation TD – DT. [5]

Group – B

(*Answer any three questions*) [3×5]

9. A particle describes an ellipse under a force which is always directed towards the centre of the ellipse. Find the law of force.

10. If h be the height attained by a particle when projected with a velocity V from the earth's surface supposing its attraction constant, and H the corresponding height when the variation of gravity is taken into account, prove that $\frac{1}{h}-\frac{1}{H}=\frac{1}{r}$, where r is the radius of the earth.

11. A heavy uniform chain of length 2ℓ, hangs over a small smooth fixed pulley, the length $\ell+c$ being at one side and $\ell-c$ at the other; if the end of the shorter portion be held and then let go, show that the chain will slip off the pulley in time $\left(\frac{\ell}{g}\right)^{1/2}\log\frac{\ell+\sqrt{\ell^2-c^2}}{c}$ $(\ell>c)$.

12. A mass is suspended from a ceiling by a light elastic string of natural length ℓ. When the mass hands in equilibrium, the length of the string is $\ell+c$. The mass is started off from this position of equilibrium with downward vertical velocity v. If in the subsequent motion the string never becomes slack, show that $v^2\le cg$.

13. A particle is projected with velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is k times the velocity. Show that its direction will again make angle α below the horizontal after a time $\frac{1}{k}\log\left(1+\frac{2ku\sin\alpha}{g}\right)$.

Group – C

(*Answer any two questions*) [2×5]

14. Prove that the lines whose direction cosines are given by the equations $a\ell+bm+cn=0$, $u\ell^2+vm^2+wn^2=0$ are perpendicular if $a^2(v+w)+b^2(w+u)+c^2(u+v)=0$ and parallel if $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$.

15. If 2c be the shortest distance between the lines $\frac{x}{\ell}-\frac{z}{n}=1$, y = 0 and $\frac{y}{m}+\frac{z}{n}=1$, x = 0, show that $\frac{1}{c^2}=\frac{1}{\ell^2}+\frac{1}{m^2}+\frac{1}{n^2}$.

16. Find the smallest sphere which touches the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z-6}{1}$ and $\frac{x+3}{7}=\frac{y+3}{-6}=\frac{z+3}{1}$.

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